4 The new Keynesian model

4.3 Solving the New keynesian model

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Monetary policy in the New keynesian model

Let us start with the three equation model:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t \tag{1}$$

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n) \tag{2}$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \upsilon_t \tag{3}$$

Monetary policy in the New keynesian model

This can be reduced to a two equation first difference model:

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} + B_T(\hat{r}_t^n - \upsilon_t) \tag{4}$$

where $\hat{r}_t^n = r_t^n - \rho$ and:

$$A_T = \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \tag{5}$$

$$B_T = \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix} \tag{6}$$

and $\Omega = \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$

Taylor principle

The Taylor principle implies that the two eigenvalues of A_T are within the unit circle. Bullard and Mitra (2002) show that:

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0 \tag{7}$$

is a necessary and sufficient condition for uniqueness.

Effects of a monetary policy shock

We are going to assume that the monetary policy shock follows an autoregressive process given by:

$$\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \epsilon_t^\upsilon \tag{8}$$

where $0 \leq \rho_\upsilon < 1$

Solving the model: the method of undetermined coefficients

This method consists in guessing a functional form for the solution. We know from the simple monetary model that a solution of the model is to express the endogenous variables as a function of the structural shocks. Note that one can solve:

$$mc = (\sigma + \frac{1+\varphi}{1-\alpha})y_t^n - \frac{1+\varphi}{1-\alpha}a_t - log(1-\alpha) \tag{9}$$

For y_t^n. Making use of $mc = -\mu$, one gets

$$y_t^n = \psi_{ya}^n a_t + \vartheta_y^n \tag{10}$$

where $\vartheta = -\frac{(1-\alpha)(\mu - log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} > 0$ and $\psi_{ya}^n = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$

Solving the model: monetary policy shock

We have seen that the natural rate of interest evolves as:

$$r_t^n = \rho + \sigma E_t \Delta y_{t+1}^n \quad (11)$$

using the equation for natural output this can be written as:

$$r_t^n = \rho + \sigma \psi_{ya}^n E_t \Delta a_{t+1} \quad (12)$$

Thus the natural rate evolution does not depend on monetary policy shocks.
If one assumes no technology shocks then: $r_t^n = \rho$

Solving the model: monetary policy shock

The solution of the model for the purposes of studying monetary policy shocks implies expressing $\tilde{y}_t$ and π_t as a function of monetary policy shocks. We already saw that the natural rate does not depend on monetary policy shocks so $\hat{r}_t^n$ is set to zero.

By the method of undetermined coefficients we are going to guess that the solution implies $\tilde{y}_t = \psi_{y\upsilon}\upsilon_t$ and $\pi = \psi_{\pi\upsilon}\upsilon_t$.

Solving the model: monetary policy shock

Let us replace the guessed solution in the IS and the New Keynesian Phillips curve:

$$\psi_{y\upsilon}\upsilon_t = E_t\psi_{y\upsilon}\upsilon_{t+1} - \frac{1}{\sigma}(i_t - E_t\psi_{\pi\upsilon}\upsilon_{t+1} - \rho) \quad (13)$$

$$\psi_{\pi\upsilon}\upsilon_t = \beta E_t\psi_{\pi\upsilon}\upsilon_{t+1} + \kappa\psi_{y\upsilon}\upsilon_t \quad (14)$$

and making use of the monetary policy rule:

$$i_t = \rho + \phi_\pi\psi_{\pi\upsilon}\upsilon_t + \phi_y\psi_{y\upsilon}\upsilon_t + \upsilon_t \quad (15)$$

Solving the model: monetary policy shock

Solving the system one gets:

$$\tilde{y}_t = -(1 - \beta\rho_\upsilon)\Lambda_\upsilon \upsilon_t \quad (16)$$

$$\pi_t = -\kappa\Lambda_\upsilon \upsilon_t \quad (17)$$

where $\Lambda_\upsilon = \frac{1}{(1-\beta\rho_\upsilon)[\sigma(1-\rho_\upsilon)+\phi_y]+\kappa(\phi_\pi-\rho_\upsilon)}$

Solving the model: technology shock

The real rate in deviations from steady state is given by:

$$\hat{r}_t = \sigma(1 - \rho_\upsilon)(1 - \beta\rho_\upsilon)\Lambda_\upsilon \upsilon_t \tag{18}$$

In turn the nominal rate is given by (in deviations from steady state):

$$\hat{i}_t = \hat{r}_t + E_t \pi_{t+1} = [\sigma(1 - \rho_\upsilon)(1 - \beta\rho_\upsilon) - \rho_\upsilon \kappa]\Lambda_\upsilon \upsilon_t \tag{19}$$

Solving the model for a technology shock

Assume a shock process of the form:

$$a_t = \rho_a a_{t-1} + \epsilon_t \tag{20}$$

we have already seen that:

$$r_t^n = \rho + \sigma \psi_{ya}^n E_t \Delta a_{t+1} \tag{21}$$

which implies:

$$\hat{r}_t^n = -\sigma \psi_{ya}^n + (1 - \rho_a) a_t \tag{22}$$

Solving the model for a technology shock

Now assume $v_t = 0$. It can be shown that:

$$\tilde{y}_t = (1 - \beta\rho_a)\Lambda_a \hat{r}_t^n \quad (23)$$
$$= -\sigma\psi_{ya}^n(1 - \rho_a)(1 - \beta\rho_a)\Lambda_a a_t \quad (24)$$

and

$$\pi_t = \kappa\Lambda_a \hat{r}_t^n \quad (25)$$
$$= -\sigma\psi_{ya}^n(1 - \rho_a)\kappa\Lambda_a a_t \quad (26)$$

where: $\Lambda_a = \frac{1}{(1-\beta\rho_a)[\sigma(1-\rho_a)+\phi_y]+\kappa(\phi_\pi-\rho_a)} > 0$

Solving the model for a technology shock

Output and employment will be given by:

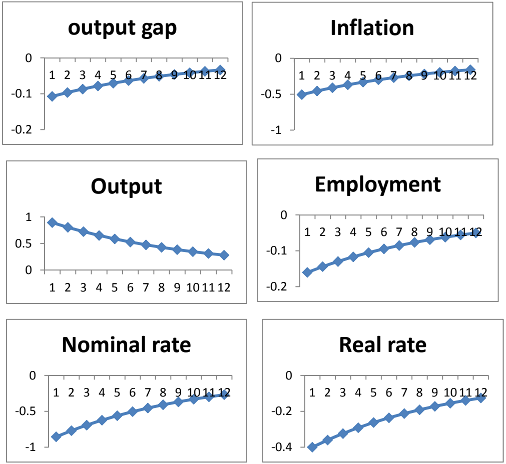
$$y_t = \psi_{ya}^n(1 - \sigma(1 - \rho_a)(1 - \beta\rho_a)\Lambda_a)a_t \tag{27}$$

$$(1 - \alpha)n_t = y_t - a_t \tag{28}$$

Model calibration

- $\sigma = 1$
- $\varphi = 1$
- $\alpha = 1/3$
- $\epsilon = 6$
- $\theta = 2/3$
- $\phi_\pi = 1.5$
- $\phi_y = 0.5/4$
- $\rho_a = 0.9$
- $\rho_\upsilon = 0.5$

Impulse responses- technology shock



Impulse responses- monetary policy shock